dsp tips & tricks

Lionel Cordesses

Direct Digital Synthesis: A Tool For Periodic Wave Generation (Part 2)

In Part 1 of this article (in the July 2004 issue of *IEEE Signal Processing Magazine*), we presented an overview of the basics of direct digital frequency synthesis (DDS), simple formulas to compute bounds of the signal characteristics, and a scheme to improve the DDS spurious free dynamic range (SFDR). In this Part 2, we discuss additional tricks used to optimize DDS performance by maximizing SFDR.

Improving SFDR Through Spur-Reduction Techniques

The easiest method to reduce the level of DDS spurs, discussed in Part 1, is to increase the accuracy of the phase to waveform converter. The limit of this approach has been mentioned; it is mainly technological [lookup table (LUT) size].

We now review three simple and effective methods to reduce the spur level of the sinewave DDS, along with the corresponding spectra computed from simulated DDS outputs.

The Odd-Number Approach

The worst-case spur level is given by (9) in Part 1. Making Δ_{ACC} an odd number improves the SFDR by 3.9 dB [14] in Part 1. The repetition period of the accumulator T_{ACC}, often referred to as the grand repetition period, is given by

$$T_{ACC} = \frac{2^N}{GCD(2^N, \Delta_{ACC})} \quad (1)$$

where $GCD(x, y)$ stands for the greatest common divisor of x and y [1]. When $GCD(2^N, \Delta_{ACC}) = 1$, as it will be whenever Δ_{ACC} is an odd number, then $T_{ACC} = 2^N$, which *spreads* the spurs over the entire spectrum (otherwise they are aliased to a frequency within the spectrum, as described in [14] and [12] in Part 1). As an example, we computed the output spectra with a fast Fourier transform, the length of which is equal to T_{ACC}, for the two cases of $\Delta_{ACC} = 13, 248$ and $\Delta_{ACC} = 13,249$, with $N = 16$, $P = 9$, and $M = 8$. The odd number for Δ_{ACC} leads to an increase of $54.2 - 50.3 = 3.9$ dB in SFDR.
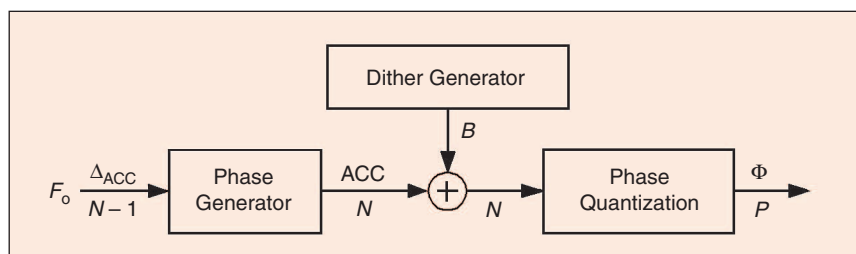
The Phase-Dithering Approach

In another method to spread the spurs throughout the available bandwidth, one can add a dither signal [2] to the ACC phase values as shown in Figure 1.

The dither signal can be a pseudo-random noise sequence (generated, for example, with binary shift registers and exclusive-or gates, and having a repetition period much greater than the output signal period) whose word width is B bits providing noise values in the range of 0 and 2^B. Choosing $B = N - P$, the spurs do follow a 12-dB per-phase bit law [3] instead of the 6-dB per-phase bit of (9), thus allowing a smaller LUT for the same SFDR.

An output spectrum is given without any dither signal in Figure 2(a) with $F_s = 44, 100$ Hz, $\Delta_{ACC} = 1,657$, $N = 16$, $P = 5$, $M = 16$, and the dither signal ($B = N - P = 16 - 5 = 11$ b) applied in Figure 2(b). The spectra have been computed for ten output signals that have been averaged following the method described in [3]. The high-resolution, 16-b, LUT has been chosen so as to focus only on the 5 b of phase quantization, as in [3]. The drawback of this dithering

▲ *1. DDS, including phase dither.*

method is the increase of the noise floor, but that's a small price to pay for such a large increase in SFDR.

Other dithering methods are available, such as amplitude dithering and both phase and amplitude dithering (see [3] and [4]). The phase-dithering approach has also been applied to squarewave signal DDS [5].

The Noise-Shaping Approach

The key idea of the noise-shaping approach, to improve our DDS SFDR, is to filter out the quantization noise introduced by the phase quantization step in Figure 3. This quantization can be viewed as a special case of noise addition [6], as depicted in Figure 3(a). The quantization noise signal n can be recovered from the following equations:

$$\Phi = n + \text{ACC}$$
$$e_Q = \text{ACC} - \Phi. \qquad (2)$$

Thus, $e_Q = -n$. In the *noise-shaping* approach, this quantization noise signal $-n$ is fed back, through a filter G, to the ACC signal as shown in Figure 3(b). The transfer function of interest is the one from n to Φ, as the noise added to the phase signal Φ will eventually lead to phase noise. The phase signal is then
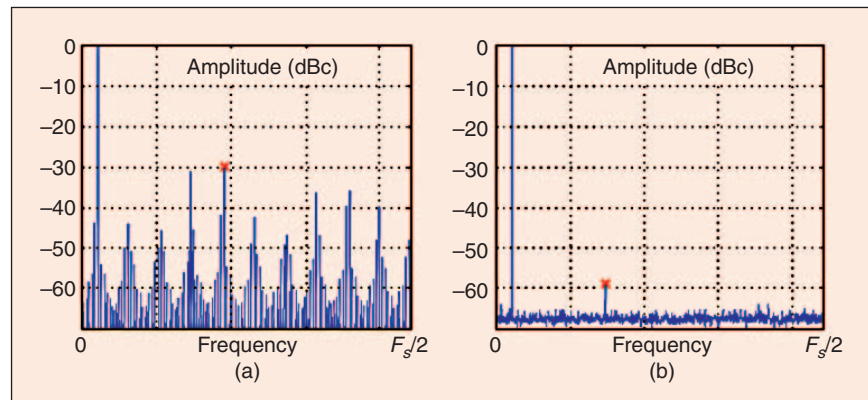
$$\Phi = n(1 - G) + \text{ACC}. \qquad (3)$$

From (3), one can infer that the phase signal Φ is affected by the filtered noise signal n, $(1-G)$ being the transfer function of the filter. The choice of G will lead to different results, as we shall see.

A *first-order noise-shaping* approach is to use the simple transfer function G proposed in [10] in Part 1 as the finite impulse response (FIR) filter $G = z^{-1}$. Here z is the symbol of the z-transform used for discrete-time systems. The function
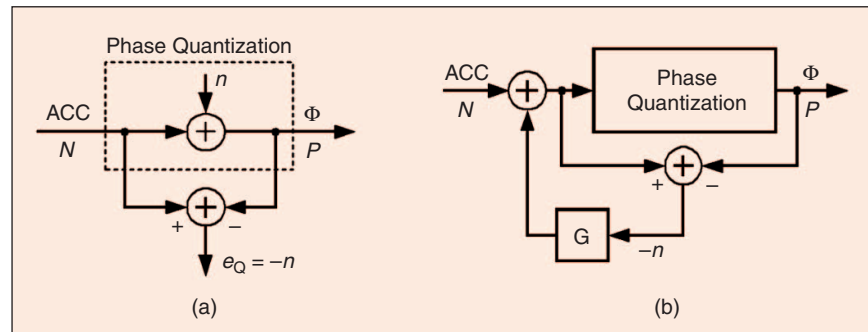
G can be implemented as a single delay register, and $1 - G$ has a 0 at $z = 0$ (0 Hz); it acts as a discrete time differentiator. The system filters out the low-frequency components of the noise signal n but high-frequency signals, greater than 8 kHz, are amplified. This simple approach prevents the filter from

rejecting high-frequency components of the noise signal n, thus justifying the statement that one should use this filter for low-frequency F_o signals [7].
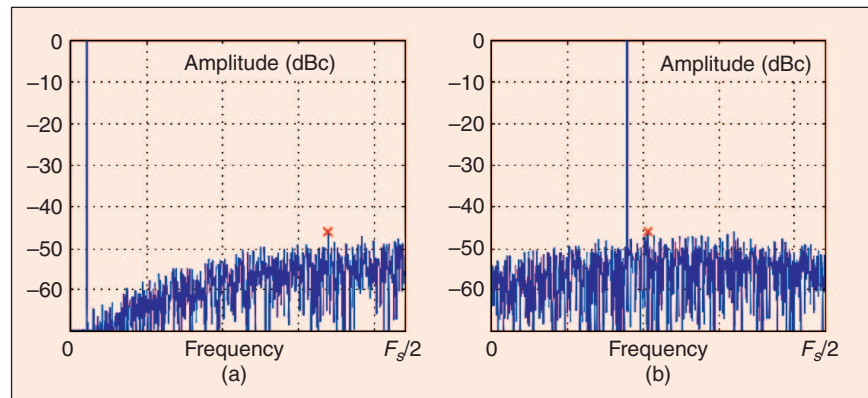
An example of the output spectrum is given in Figure 4(a). [The parameters are the same as in Figure 2(a).] The SFDR is greater

▲ 2. DDS output spectra: (a) without dither and (b) with dithering.

▲ 3. Quantization and noise shaping: (a) quantization model and (b) noise-shaping implementation.

▲ 4. DDS spectra with first-order noise shaping and $G = z^{-1}$: (a) $\triangle_{\text{ACC}} = 1,657$ and (b) $\triangle_{\text{ACC}} = 13,249$.

than 60 dB near the carrier, but it decreases with the frequency and eventually reaches 46.8 dBc. The overall behavior is compliant with the above $G = z^{-1}$ analysis. At a higher frequency ($\Delta_{ACC} = 13249$), the noise close to the carrier is less filtered, as one can see in Figure 4(b).

To implement *higher-order noise shaping*, a more complex filter can be used instead of $G = z^{-1}$. A second-order FIR filter has been proposed in [8]:

$$1 - G = 1 + b_1 z^{-1} + b_2 z^{-2}. \quad (4)$$

Careful choice of b_1 and b_2 can lead to a double zero at 0: $1 - G = 1 - 2z^{-1} + z^{-2} = (1 - z^{-1})^2$, which improves the rejection at 0 Hz. When the noise shaping is applied to the amplitude signal instead of the phase signal, other values (often integer values, so as to ease implementation, see [8]) are preferred, and this filter can even be tuned online.

Multiple-zeros filters are also of interest, for example, when one wants to reject a known frequency such as $2 \times F_o$ [7]. A tunable notch filter is added at the expense of a more complex feedback structure. The transfer function becomes

$$\begin{aligned}
1 - G &= (1 - z^{-1})(1 + bz^{-1} + z^{-2}) \\
&= 1 - (1 - b)z^{-1}(1 - b)z^{-2} \\
&\quad - z^{-3}
\end{aligned} \quad (5)$$

with $b = -2\cos[2\pi(2F_o/F_s)]$. At low frequencies, as shown in Figure 5(a), the spectrum looks like the one with the first-order noise shaping. But at a higher frequency [the same as in Figure 4(b)], the improvement close to the carrier is clear, as shown in Figure 4(b).

Note that the same filter (with zero at a specific frequency) can be implemented by feeding the error signal back to the accumulator. This structure, proposed and patented in [7], is presented in Figure 6. F is the transfer function of the accumulator [an integrator: $ACC(k) = ACC(k-1) + \Delta_{ACC}(k-1)$], given by

$$F = \frac{z^{-1}}{1 - z^{-1}} \quad (6)$$

with the output phase Φ given by $\Phi = (1 - FG)n + F\Delta_{ACC}$. One can chose G so as to ensure $(1 - FG) = (1 - z^{-1})(1 + bz^{-1} + z^{-2})$ as in (5).
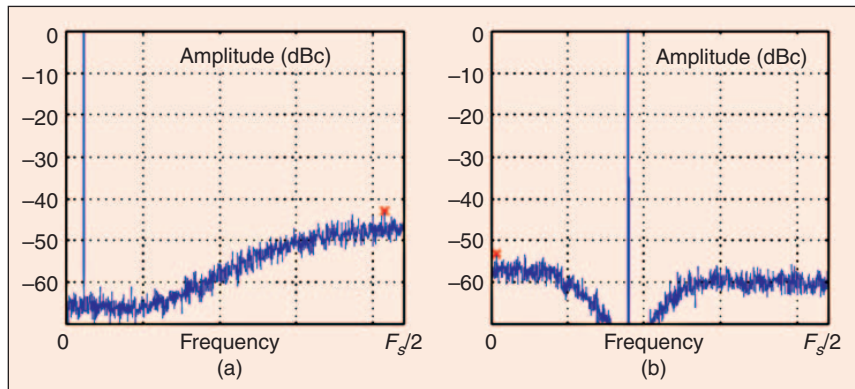
Summary

DDS is a useful tool for generating periodic waveforms. In this two-part article, we presented the basic idea of this synthesis technique and then focused on the quality of the sinewave a DDS can create, introducing the SFDR quality parameter. Next we presented effective methods to increase the SFDR through sinewave approximations, hardware schemes such as dithering and noise shaping, and an extensive list of references. When the desired output is a digital signal, the signal's characteristics can be accurately predicted using the formulas given in this article. When the desired output is an analog signal, the reader should keep in mind that the performance of the DDS is eventually limited by the performance of the digital-to-analog converter and the follow-on analog filter [9].

We hope that this article will incite engineers to use DDS; either integrated-circuits DDS or software-implemented DDS. From the author's experience, this technique has proved valuable when frequency resolution is the challenge, particularly when using low-cost microcontrollers.
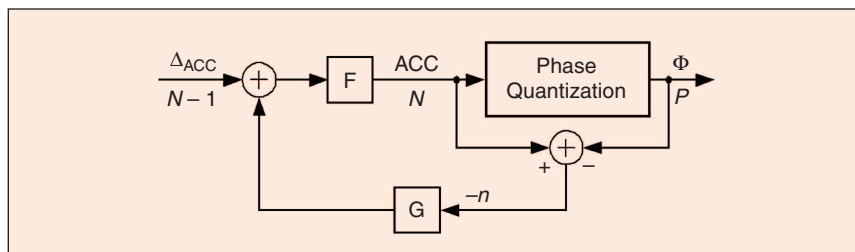

▲ 5. DDS spectra with higher-order noise shaping: (a) same parameters as Figure 2(a) and (b) same parameters as Figure 4(b).

▲ 6. Noise shaping: Analog Devices' approach.

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Anritsu Company, Corporate Offices, 490 Jarvis Dr., Morgan Hill, CA 95037-2809 USA.
Tel: +1 800 267 4878
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Lionel Cordesses is a control engineer at the Techno-centre Renault (Guyancourt, France), the research center of the Renault company. His technical interests include analog electronics, digital signal processing, and embedded controllers. He is a member of the IEEE and the ACM.

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What's Next?

Here's what's planned for future issues:

• Color Image Processing

• Signal Processing for Surveillance Networks